Time Frame: 50 minutes

Subject Matter: Discrete Probability Distribution

TELL ME

Objective: TSWBAT Find the expected value for a discrete random variable.

Standards: DA – 5.11

 Materials: Transparencies and Worksheets

SHOW ME

Presentation of Information:

The teacher will discuss the following:

* Expected Value

The expected value of a discrete random variable of a probability distribution is the theoretical average of the variable. The formula is

$$μ=E\left(x\right)= ΣX∙P(X)$$

* + This can be used in various types of games of chance, in insurance, and in other areas, such as decision theory.

Example 1

A ski resort loses $70,000 per season when it does not snow very much and makes $250,000 profit when it snows a lot. The probability of its snowing at least 75 inches (i.e., in good season) is 40%. Find the expectation for the profit.

Solution:

Profit X ­­­$250,000 -$70,000­\_\_\_­­

Probability P(X) 0.40 0.60

*E(X)* = ($250,000)(0.40) + (-$70,000)(0.60)

*E(X)* = $58,000

Example 2

One thousand tickets are sold at $1 each for four prizes of $100, $50, $25, and $10. What is the expected value if a person purchases two tickets?

Solution:

Gain X $98 $48 $23 $8 -$2\_\_\_­­

Probability P(X) $\frac{2}{1000}$ $\frac{2}{1000}$ $\frac{2}{1000}$ $\frac{2}{1000}$ $\frac{992}{1000}$

$$E\left(X\right)=\$98∙\frac{2}{1000}+ \$48∙\frac{2}{1000}+\$23∙\frac{2}{1000}+\$8∙\frac{2}{1000}+-\$2∙\frac{992}{1000}$$

$$E\left(X\right)=-\$1.63$$

This means that if a person purchased 2 tickets over a long period of time, the average loss would be $1.63.

* In gambling games, if the expected value of the game is zero, the game is said to be fair.
* If the expected value of the game is positive, the game is in favor of the player. That is, the player has a better-than-even chance of winning.
* If the expected value of the game is negative, then the game is in favor of the house. That is, in the long run, the players will lose money.
* In his book *Probabilities in Everyday Life* (Ivy Books, 1986), author John D. McGervy gives the expectations for various casino games. For keno, the “house” wins $0.27 on every $1 bet. For chuck-a-luck, the house wins about $0.52 on every $1 bet. For roulette, the house wins about $0.90 on every $1 bet. For craps, the house wins about $0.88 on every $1 bet.
* The bottom line here is that if you gamble long enough or later, you will end up losing money.

Let Me Try

1. The Lincoln Fire Department wishes to raise $5,000 to purchase some new equipment. They decide to conduct a raffle. A cash prize of $5,000 is to be awarded. If 2500 tickets are sold at $5 each, find the expected value of the gain. Are they selling enough tickets to make their goal?
2. A box contains ten $1 bills, five $2 bills, three $5 bills, one $10 bill, and one $100 bill. A person is charged $20 to select one bill. Find the expectation. Is the game fair?
3. If a person rolls doubles when he tosses two dice, he wins $5. For the game to be fair, how much should the person pay to play the game?

Homework:

* If a player rolls two dice and gets a sum of 2 or 12, she wins $20. If the person gets a 7, she wins $5. The cost to play the game is $3. Find the expectation of the game.